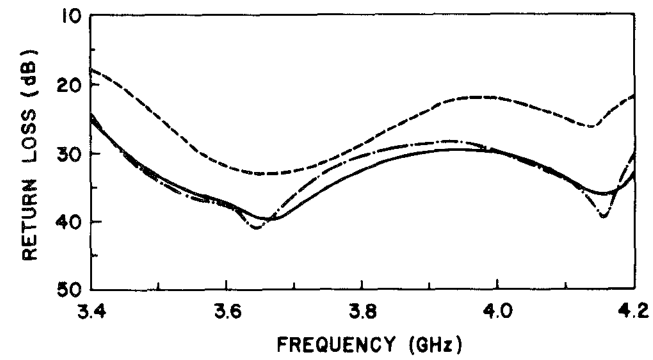

Fig. 4. Return loss, as a function of frequency, for the mode converter shown in Fig. 3. (———) Theoretical results according to the present method; (—·— —) experimental results; (----) theoretical results according to [1].

converters terminated into corrugated waveguides with shallow corrugations.

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Resonant Frequency of Cylindrical Dielectric Resonator Placed in an MIC Environment

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***Abstract*—In this paper, an effective dielectric constant technique has been used to determine the resonant frequency of the $TE_{01\delta}$ mode of a cylindrical dielectric resonator placed in an MIC environment. A suitable expression for ϵ_{eff} has been reported which makes it possible to obtain results that compare favorably with rigorous methods. A large number of experimental results are also reported to demonstrate the validity of the method. Finally, for a given resonant frequency, closed-form expressions are given for computing the height of the resonator.**

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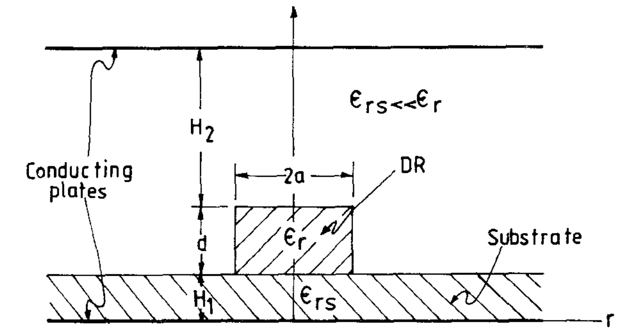

Fig. 1. Cylindrical dielectric resonator placed in an MIC environment.

I. Introduction

A typical configuration in which a cylindrical dielectric resonator is used in MIC's is shown in Fig. 1. A few rigorous methods have been reported during the last few years for determining the resonant frequency of a dielectric resonator placed in configurations of the type shown in Fig. 1 (e.g. [1] and [2]). A number of rigorous methods have also been reported for structures which are special cases of the structure shown in Fig. 1 [3]–[9]. However, all the rigorous methods are computationally quite complex, which makes their use in practical design applications almost prohibitive. On the other hand, approximate methods such as the dielectric waveguide model (DWM) method [10] are simple to use but do not offer adequate accuracy.

Today's CAD trends indicate the need for a method which offers both simplicity and accuracy. An approximate but accurate and simple effective dielectric constant technique has previously been proposed for the analysis of isolated cylindrical dielectric resonators [11], [12]. The technique is basically an improvement of the DWM method. In principle, the improvement is similar to that offered by the EDC technique [13] over Marcatili's method [14] for analyzing rectangular dielectric waveguide structures. In this paper, we use the effective dielectric constant technique to find the resonant frequency of the structure shown in Fig. 1. The technique leads to results nearly matching in accuracy those of rigorous methods. The resonant frequency of the resonator has been obtained by using a suitable approximation for ϵ_{eff}. The expression reported for ϵ_{eff} is different from the one used earlier for the case of an isolated resonator [11], [12].

II. Theory

We limit our attention to the lowest order $TE_{01\delta}$ mode of resonance, which is the most commonly employed mode of resonance in practical applications. For this mode, only three field components, i.e., E_ϕ, H_z and H_r, exist. The H_z component, from which the other field components can be derived, is assumed to be of the following form inside the resonator at resonance:

$$H_z = J_0(hr)\left[A_m \cos\{\beta(z-H_1)\} + B_m \sin\{\beta(z-H_1)\}\right]. \quad (1)$$

In the above equation J_0 denotes the Bessel function of first kind and order zero. The problem of finding the resonant frequency is one of finding wavenumbers h and β which also satisfy the separation equation

$$h^2 + \beta^2 = \epsilon_r k_0^2 \quad (2)$$

where k_0 is the free-space wavenumber corresponding to the

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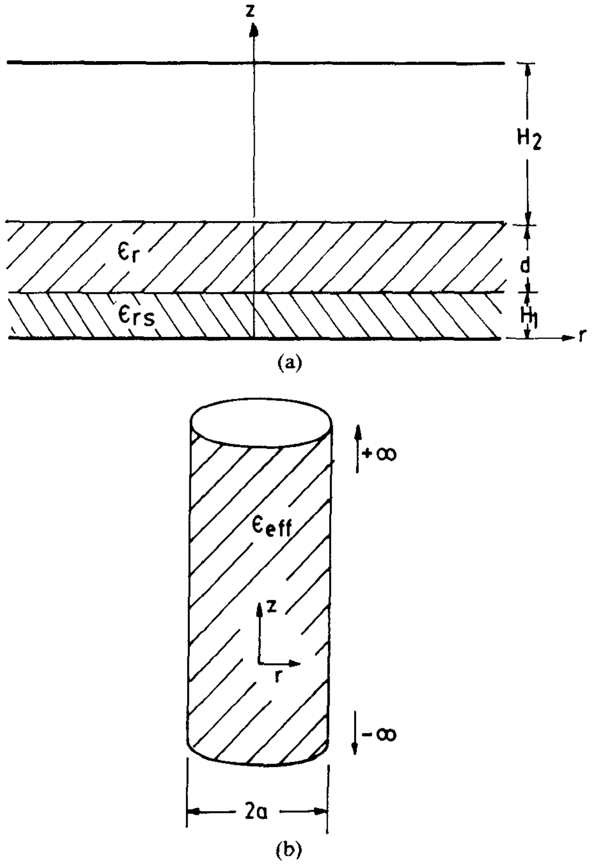

Fig. 2. (a) Double layer radial slab-guide model for determining wavenumber β. (b) Infinite cylindrical dielectric waveguide model for determining radial wavenumber h of the resonator.

resonant frequency. The method of determining wavenumbers β and h is now described.

A. Determination of Axial Wavenumber β

The DWM method is used to determine the axial wavenumber β. It is assumed that the axial wavenumber β is same as that of the double-layer radial slab-guide structure obtained by extending the resonator to infinity in the radial direction. The structure is shown in Fig. 2(a). The characteristic equation for the lowest order TE mode of the slab-guide structure is given by [10]

$$\beta d = \tan^{-1}\left[\left(\frac{\alpha}{\beta}\right)\coth(\alpha H_1)\right] + \tan^{-1}\left[\frac{\gamma}{\beta}\coth(\gamma H_2)\right] \quad (3a)$$

where

$$\alpha = \left[(\epsilon_r - \epsilon_{rs})k_0^2 - \beta^2\right]^{1/2} \quad (3b)$$

$$\gamma = \left[(\epsilon_r - 1)k_0^2 - \beta^2\right]^{1/2}. \quad (3c)$$

B. Determination of Radial Wavenumber h

It is assumed that the radial wavenumber h is the same as that of the TE_{01} mode of an infinitely long cylinder having the same radius as that of the resonator but having a dielectric constant ϵ_{eff} which is less than that of the resonator (see Fig. 2(b)). In the DWM method it is assumed that the dielectric constant of the infinite cylinder is the same as that of the resonator ($\epsilon_{eff} = \epsilon_r$). The transcendental equation for the radial wavenumber h is as follows:

TABLE I
RESONANT FREQUENCY (f_0) OF THE $TE_{01\delta}$ MODE OF A CYLINDRICAL DIELECTRIC RESONATOR PLACED IN AN MIC ENVIRONMENT

Structural parameters						Resonant frequency (GHz)		
ϵ_r	ϵ_{rs}	a (mm)	d (mm)	H_1 (mm)	H_2 (mm)	Rigorous theory [Ref]	DWM method	Present theory
38.0	1.0	15.00	15.00	0	∞	1.907 [5]	1.98	1.918
38.0	1.0	2.86	2.38	0	∞	10.90 [7]	11.23	10 92
						10.79 [6]		
37.4	1.0	4.25	3.31	0	∞	7.56 [8]	7 85	7 63
34.6	1.0	9.00	13.37	0	∞	2.89 [9]	2.98	2.89
						2.86 [2]		
38.0	1.0	2.86	2.38	0.72	∞	9.83 [6]	10.23	9.83
						10.00 [7]		
38.0	1.0	2.86	2.38	2.86	∞	9.16 [6]	9.86	9.14
						9.25 [7]		
35.2	9.6	2.43	1.81	0.64	∞	12.40 [3]	12.89	12.37
34.19	9.6	7.49	7.48	0.70	0.72	4.35 [1]	4.36	4.34
34.21	9.6	7.00	6.95	0.70	1.25	4.52 [1]	4.53	4 51
36.2	1.0	2.03	5.15	2.93	2.93	10.50 [4]	10.86	10.37
36.2	1.0	4.00	2.14	4.43	4.43	7.76 [4]	8.38	7.69

$$\frac{J_1(ha)}{hJ_0(ha)} + \frac{K_1(pa)}{pK_0(pa)} = 0 \quad (4a)$$

where

$$p = \left[(\epsilon_{eff} - 1)k_0^2 - h^2\right]^{1/2}. \quad (4b)$$

A suitable approximation for ϵ_{eff} is as follows:

$$\epsilon_{eff} = (\epsilon_{eff\,a} + \epsilon_{eff\,b})/2 \quad (5a)$$

where

$$\begin{aligned} \epsilon_{eff\,a} &= \epsilon_r - (\epsilon_r - \epsilon_{eff0})H_1/a, & H_1/a \leq 1 \\ &= \epsilon_{eff0}. & H_1/a > 1 \quad (5b) \\ \epsilon_{eff\,b} &= \epsilon_r - (\epsilon_r - \epsilon_{eff0})H_2/a, & H_2/a \leq 1 \\ &= \epsilon_{eff0}, & H_2/a > 1. \quad (5c) \end{aligned}$$

In (5b) and (5c),

$$\epsilon_{eff0} = q^2/k_0^2 \quad (5d)$$

where q is the radial wavenumber of an infinitely long cylinder whose radius is the same as that of the resonator.

When $H_1 = H_2 = 0$, the value of ϵ_{eff} reduces to ϵ_r. The method thus gives the exact value of the resonant frequency for a dielectric post resonator. When both H_1 and H_2 tend to infinity, moreover, ϵ_{eff} reduces to q^2/k_0^2, which is the expression that has been used previously for an isolated resonator [11], [12]. The expression given by (5a) is thus more general and is introduced to take into account the effect of top and bottom conducting planes.

C. Resonant Frequency

The resonant frequency f_0 is determined so that the computed wavenumbers h and β also satisfy (2) simultaneously.

III. THEORETICAL RESULTS

The accuracy of the present simple method is first illustrated by comparing the results for the resonant frequency with those computed using rigorous methods. The results were computed

TABLE II
COMPARISON OF THEORY WITH EXPERIMENT
($a = 4.55$ MM, $\epsilon_r = 37.1$, $H_2 \to \infty$)

H_1 (mm)	ϵ_{rs}	H_1/a	Resonant frequency (GHz) d = 2.00mm Exp.	d = 2.00mm Theory	d = 4.04mm Exp.	d = 4.04mm Theory	d = 6.96mm Exp.	d = 6.96mm Theory
0.00	-	.00	9.70	9.87	6.69	6.74	5.50	5.47
0.38	2.22	.08	8.81	8.90	6.46	6.47	5.43	5.37
0.64	9.60	.14	8.40	8.44	6.31	6.31	5.37	5.31
0.80	2.22	.17	8.30	8.27	6.30	6.24	5.34	5.28
1.58	2.22	.35	7.71	7.70	6.02	6.00	5.25	5.17
2.38	2.22	.52	7.39	7.44	5.91	5.87	5.17	5.10
3.16	2.22	.69	7.30	7.30	5.82	5.79	5.14	5.05
3.96	2.22	.87	7.18	7.19	5.78	5.74	5.10	5.02

for structural parameters for which results computed using rigorous methods are available in the literature. Table I shows the comparison. It is interesting to note from the table that the present effective dielectric constant technique gives results having an accuracy comparable to that of the rigorous methods. The difference between the results of the present theory and the rigorous methods is generally less than 1%. It should be noted that in Table I we make comparisons over a wide range of structural parameters.

The results obtained using the DWM method are also shown in Table I to demonstrate the improvement the present method offers. It is seen that when both H_1/a and H_2/a are small, the error of the DWM method in predicting the resonant frequency is quite small. Otherwise, the error of the DWM method can be quite large.

IV. EXPERIMENTAL RESULTS

For experimental verification of the present theory, resonant frequencies were measured for various combinations of resonator and substrate dimensions. Resonant frequency measurements were made by placing a resonator on top of the substrate and exciting the $TE_{01\delta}$ mode by means of a loop. The results are presented in Table II along with the results from the present theory. A comparison of the theoretical and the experimental results shows that agreement is good in all cases. The difference between the results of theory and those of experiment is less than 1% in most cases. Further, in no case does this difference exceed 2%.

V. CLOSED-FORM EXPRESSIONS

For given resonator parameters f_0, ϵ_r, ϵ_{rs}, a, H_1, and H_2, a quite accurate closed-form expression can be derived for the resonator height d. The transcendental equation (4a) can be replaced by the following approximate but quite accurate simple closed-form expression [15]:

$$ha = .951p_{01} + 0.222\left[(\epsilon_{\text{eff}} - 1)(k_0 a)^2 - .951p_{01}^2\right]^{1/2} \quad (6a)$$

where $p_{01} = 2.405$ is the first root of the equation $J_0(x) = 0$. Similarly, the value of q which is required for determining ϵ_{eff} can be obtained using the following closed-form expression:

$$qa = .951p_{01} + 0.222\left[(\epsilon_r - 1)(k_0 a)^2 - .951p_{01}^2\right]^{1/2}. \quad (6b)$$

Once h is known, the values of β, α, and γ are determined using (2), (3b), and (3c) respectively. Further, using (3a) the resonator length d can be obtained.

Alternatively, if ϵ_r, ϵ_{rs}, a, d, H_1, and H_2 are known, the above procedure can be used in an iterative manner to determine the resonant frequency.

VI. CONCLUSIONS

In this paper, an effective dielectric constant technique has been used to compute the resonant frequency of the $TE_{01\delta}$ mode of a dielectric resonator placed in an MIC environment. A suitable expression for ϵ_{eff} has been reported. A large number of experimental results are also reported to demonstrate the validity of the present method. The results amply demonstrate the usefulness of the method. It is important to remark here that the present method is useful for determining not only the resonant frequency, but also the field distribution inside the resonator through (1). This information can be used to derive other quantities, such as stored energy, fields outside the resonator, and different Q factors, quite accurately [16].

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